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Local energy minima in quantum spin glasses

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Abstract. The eigenstates of a six-site cluster with spins of $S=\frac{1}{2}$ coupled by Heisenberg interactions are studied exactly. We have investigated various criteria (variational, perturbational and related to a stability of 'spin waves') to distinguish a local energy minimum, if any, from ordinary quantum states. We have found that a sensible definition of a local energy minimum is similar to the classical one. It involves investigating the stability of the system against infinitesimal spin rotations. We give an example of a six-spin system with three minima which are stable against single-spin rotations of which two seem to be stable against simultaneous many-spin rotations.

1. Introduction

The central idea in spin-glass physics (Binder and Young 1986) is that of multiple local energy minima. A spin glass placed in any such minimum would stay there indefinitely if there was no heat bath surrounding it. At any finite T, however, the system is characterised by peculiar dynamical effects which may last on timescales of hours or even days.

A model which embodies the essentials of this physics has been proposed by Edwards and Anderson (1975). In this model, the Hamiltonian is given by

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \tag{1}$$

where J_{ij} are Gaussian random numbers. The spin-spin interactions in equation (1) are written in the Heisenberg form but a related model, in which the interactions are of the Ising type, has been studied even more extensively.

In the case of Heisenberg spins research has mainly concentrated on the classical limit of the model. In this limit, one considers the spins to be vectors which can take any direction on a sphere. The concept of an energy minimum is well defined here (see, e.g., Walker and Walstedt 1980). A minimum corresponds to such a spin configuration that any infinitesimal departure away from it induces an increase in energy. In order to find the minima one usually starts from some random spin configuration and aligns spins sequentially towards their local exchange fields until spins cease adjusting. A state uniformly rotated in space is considered to be equivalent to an unrotated one. Note that the alignment is a spin by spin process, so that a stability against multiple-spin deviations is not necessarily guaranteed.

In the case of Ising spins a local energy minimum is defined in a similar fashion. The only difference is that instead of infinitesimal rotations one studies spin reversals. Again, one usually investigates only single-spin stability which relates to a particular choice of the spin dynamics—that proposed by Glauber (1963, see also Banavar *et al* 1985, Cieplak and Łusakowski 1986).

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The Ising and classical Heisenberg spin glasses seem to be well investigated by now. Very little is known, however, about quantum effects in frustrated systems. Some pioneering studies have been undertaken (Fisher 1975, Bray and Moore 1980, Theumann 1986). However, they have not yet elucidated the meaning of the concept of a local energy minimum—other than the ground state—in quantum systems.

In this paper we attempt to clarify this notion by studying the six-spin cluster of figure 1 where the spins are the $S=\frac{1}{2}$ quantum objects and the exchange couplings are as indicated. Each site has a coordination number of four. An equivalent geometry is that of a ring of six spins with first- and second-neighbour couplings. The interactions are described by equation (1) where $\boldsymbol{\sigma}_i$ now consists of the three Pauli matrices. The merit of studying this system is that it allows for an exact analysis of its eigenfunctions. There are altogether only 64 states in this small cluster but in a certain sense, as we shall see, some of them can be considered as corresponding to two different local energy minima. Another interest in studying this system is that its classical counterpart—exchangewise—has only one minimum. Note, however, that even the ground states of the two systems cannot be compared to each other meaningfully due to the fact that a continuum of classical states has no correspondence to the 64 states of the spin-$\frac{1}{2}$ system. Recall that the classical limit corresponds to taking $S\to\infty$, $\hbar\to 0$ with their product staying constant.

The properties of quantum antiferromagnetic rings of several spins with nearest-neighbour couplings have been discussed by Orbach (1959). The studies reported here can be thought of as an extension of the Orbach work to spin glasses. Our main

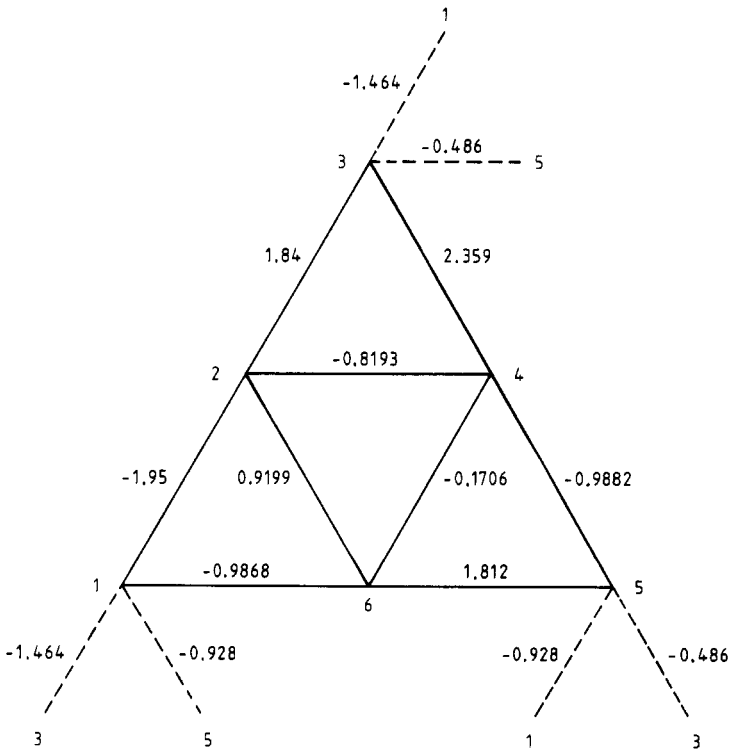

Figure 1. The six-spin cluster considered in this paper. The numbers indicate values of the exchange couplings for the spin-glass case.

concern here is how to distinguish local energy minima, if any, from ordinary quantum states.

2. Eigenfunctions

Consider the cluster of figure 1. Its corresponding Hamiltonian (1) is a 64×64 matrix. We choose the basis to consist of states of definite z spin components at individual sites. It is convenient to introduce

$$\boldsymbol{S} = \tfrac{1}{2}\hbar \sum_k \boldsymbol{\sigma}_k. \tag{2}$$

Thus all of the eigenstates $|E, S, m\rangle$ of $\mathscr{H}$ can be enumerated by the energy and the quantum numbers of $\boldsymbol{S}^2$ and S^z respectively. For the six-spin cluster the following multiplets are allowed: (i) a single multiplet corresponding to $S = 3$, (ii) five multiplets with $S = 2$, (iii) nine with $S = 1$, and (iv) five with $S = 0$. It follows that there are 20 possible energy eigenvalues here (in the case of Ising spins there would be 32).

In the ferromagnetic case $J_{ij} = 1$ there is a significant degeneracy since we have six different energy eigenvalues. The energies of the multiplets are then as follows:

$S = 3$: -12

$S = 2$: -4, -4, -4, 0, 0

$S = 1$: 0, 0, 0, 4, 4, 4, 8, 8, 8

$S = 0$: 0, 8, 8, 8, 12.

The ground state $|g\rangle$ is a septuplet corresponding to $E = -12$.

In the spin-glass case, with J_{ij} as in figure 1, the degeneracy is removed and the energies are as follows:

$S = 3$: 0.862

$S = 2$: -12.691, -4.006, 0.836, 6.356, 10.366

$S = 1$: -15.492, -8.685, -5.476, -4.636, -2.708, 3.729, 5.218, 9.098

$S = 0$: -14.280, -5.360, -1.825, 5.776, 13.104, 16.366.

The ground state $|E_0, 1\rangle$ is a triplet corresponding to $E_0 = -15.492$. Its highest m component is

$$\begin{aligned}|E_0, 1, 1\rangle = {} & -0.0886|\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\rangle + 0.2043|\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\rangle + 0.6200|\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow\rangle - 0.4894|\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\rangle \\ & -0.2463|\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle - 0.1083|\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\rangle - 0.2209|\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\rangle \\ & +0.2647|\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\rangle + 0.1531|\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\rangle - 0.2124|\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\rangle \\ & +0.1006|\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\rangle + 0.1011|\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\rangle + 0.0158|\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle \\ & -0.0700|\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\rangle - 0.1167|\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\rangle + 0.1941|\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle.\end{aligned}$$

Its $m = 0$ component is

$$\begin{aligned}|E_0, 1.0\rangle = {} & a_1(|\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\rangle + |\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\rangle) + a_2(|\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\rangle) \\ & + a_3(|\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\rangle) + a_4(|\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\rangle + |\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\rangle) \\ & + a_5(|\downarrow\uparrow\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\rangle) + a_6(|\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle) \\ & + a_7(|\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle| + a_8(|\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\rangle\rangle + |\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow\rangle) \\ & + a_9(|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\rangle) + a_{10}(|\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle)\end{aligned}$$

where $a_1 = 0.0052$, $a_2 = 0.2196$, $a_3 = -0.2215$, $a_4 = -0.1286$, $a_5 = 0.4327$, $a_6 = -0.1304$, $a_7 = -0.0185$, $a_8 = 0.0428$, $a_9 = 0.1817$ and $a_{10} = -0.3831$.

The first excited state $|E_1, 0\rangle$ is a singlet of energy $E_1 = -14.280$. The corresponding wavefunction is similar to $|E_0, 1, 0\rangle$ with symmetric combinations ($|\ \rangle + |\ \rangle$) replaced by antisymmetric ones ($|\ \rangle - |\ \rangle$) and with $a_1 = 0.1066$, $a_2 = -0.0636$, $a_3 = -0.0435$, $a_4 = 0.0004$, $a_5 = -0.4190$, $a_6 = 0.1642$, $a_7 = 0.1482$, $a_8 = 0.2553$, $a_9 = 0.2273$ and $a_{10} = -0.3760$.

At this point it is convenient to describe the properties of some related models in which the exchange couplings are as in figure 1. The classical Heisenberg system has one minimum only (4^3 systems or so start yielding more than one minimum). We started from more than 300 initial configurations of unit vectors and kept minimising the energy. Each time we got an energy of -10.8198 and a magnetisation $|\boldsymbol{M}| = 2.49$. The ground state is certainly stable against multiple-spin deviations. The Ising system ($\sigma_i = \pm 1$), on the other hand, is characterised by two energy minima which are stable against single-spin reversals. Each of these minima is a doublet. The ground state corresponds to $|\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$ and to its inverted image, both yielding an energy of -9.80. The higher minimum is at $E = -7.42$ and it corresponds to $|\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle$ or $|\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle$. This minimum is not stable against a simultaneous reversal of two spins.

Finally, it is proper to make a comment about the choice of the exchange couplings defining the model of figure 1. The 12 couplings shown come from a Gaussian random number generator. The only relevance of this particular choice is that it offers a possibility to study a system with more than one local energy minimum. At the same time it is small enough to allow for an exact study. Other choices of J_{ij} will surely yield different numbers of the minima. For instance, the couplings which differ in sign to those shown in figure 1 yield only one minimum: the ground state. This is in contrast to the fact that the Ising counterpart of the system considered in Banavar *et al* (1985) and Cieplak and Łusakowski (1986) has three such minima. It seems worthwhile to investigate the statistics of local energy minima in ensembles of quantum systems with Gaussian couplings as a function of the number of spins and the dimensionality.

3. Energy minimalisation by variational methods

We now return to the cluster of quantum spins and ask which of the 20 multiplets behave like minima. One way to define a minimum could be as follows. Suppose we choose some set of states and construct a wavefunction $|\psi\rangle$ as a linear combination of the states within the set. For simplicity, let the states be some selected energy eigenfunctions $|e_\lambda\rangle$. Thus

$$|\psi\rangle = \sum_\lambda (x_\lambda + \mathrm{i}y_\lambda)|e_\lambda\rangle \tag{3}$$

with

$$\sum_\lambda (x_\lambda^2 + y_\lambda^2) = 1 \tag{4}$$

to ensure the proper normalisation. This condition means that

$$x_1^2 = 1 - \sum_{\lambda \neq 1} x_\lambda^2 - \sum_\lambda y_\lambda^2 \tag{5}$$

where $|e_1\rangle$ is an arbitrarily chosen state. We now envisage some sort of $T = 0$ Monte Carlo process in which one starts from a set of x_λ and y_λ and keeps modifying the coefficients so that the average energy

$$E = \sum_\lambda (x_\lambda^2 + y_\lambda^2) e_\lambda \tag{6}$$

diminishes, subject to condition (5). Combining (5) and (6) we obtain

$$E = e_1 + \sum_{\lambda \neq 1} x_\lambda^2 (e_\lambda - e_1) + \sum_\lambda y_\lambda^2 (e_\lambda - e_1). \tag{7}$$

It follows that

$$\frac{\partial E}{\partial x_\lambda} = 2(e_\lambda - e_1) x_\lambda \qquad \lambda \neq 1 \tag{8}$$

$$\frac{\partial E}{\partial y_\lambda} = 2(e_\lambda - e_1) y_\lambda. \tag{9}$$

A necessary condition for an extremum is $x_\lambda = y_\lambda = 0$ for $\lambda \neq 1$ (or more generally, for λ outside the degeneracy subspace corresponding to e_1). If $e_1 < e_\lambda$ for each $\lambda \neq 1$ then $|e_1\rangle$ is a (global) minimum. This means that in the Monte Carlo process the lowest energy state available is approached. If the ground state was within the set, the system would land in it whatever the properties of the other states.

We conclude that the concept of a local minimum is more subtle and must involve variations within such sets of states which are restricted by something else than energy. One example of such restrictions will be discussed in the next section.

4. Stability against infinitesimal rotations

We now investigate directional variations of the energy eigenstates and envisage spins rotating infinitesimally around random axes. A state of locally minimal energy should gain energy in the process.

The six-spin rotation can be characterised by six local axes, as determined by six versors $\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_6$, and by six angles of rotation, $\phi_1, \ldots, \phi_6$. Let

$$R = \exp(-\tfrac{1}{2}\mathrm{i}\boldsymbol{n}_1 \cdot \boldsymbol{\sigma}_1 \phi_1) \exp(-\tfrac{1}{2}\mathrm{i}\boldsymbol{n}_2 \cdot \boldsymbol{\sigma}_2 \phi_2) \ldots \exp(-\tfrac{1}{2}\mathrm{i}\boldsymbol{n}_6 \cdot \boldsymbol{\sigma}_6 \phi_6). \tag{10}$$

The wavefunction

$$\psi = R|e_\lambda\rangle \tag{11}$$

corresponds to a state rotated away from a given energy eigenstate $|e_\lambda\rangle$. The rotated state admixes other energy eigenstates resulting in a modified expectation value, E_R, of the energy. E_R is given by

$$E_R(\lambda) = \langle e_\lambda | R^+ \mathscr{H} R | e_\lambda \rangle. \tag{12}$$

It is straightforward to show that

$$\left.\frac{\partial E_R(\lambda)}{\partial \phi_k}\right|_{\{\phi_k = 0\}} = \tfrac{1}{2}\mathrm{i}\langle e_\lambda | [\boldsymbol{n}_k \cdot \boldsymbol{\sigma}_k, \mathscr{H}] | e_\lambda \rangle \tag{13}$$

$$A_{kl}(\lambda) = \left.\frac{\partial^2 E_R(\lambda)}{\partial \phi_k \, \partial \phi_l}\right|_{\{\phi_k = 0\}} = \tfrac{1}{4}\langle e_\lambda | [[\mathscr{H}, \boldsymbol{n}_l \cdot \boldsymbol{\sigma}_l], \boldsymbol{n}_k \cdot \boldsymbol{\sigma}_k] | e_\lambda \rangle. \tag{14}$$

The first derivative at $\{\phi_k = 0\}$ vanishes for any $|e_\lambda\rangle$. An infinitesimal rotation would be considered stable if the 6×6 matrix $A_{kl}(\lambda)$ of the second derivatives was positive definite. The corresponding state would be a local minimum.

The matrix of the second derivatives is a function of twelve angular parameters which specify the directions $\boldsymbol{n}_1, \ldots, \boldsymbol{n}_6$. The matrix can be rewritten in the following form:

$$A_{kl}(\lambda) = -\delta_{kl} \sum_{p \neq l} J_{lp} \langle e_\lambda | (\boldsymbol{n}_l \cdot \boldsymbol{\sigma}_l)(\boldsymbol{n}_l \cdot \boldsymbol{\sigma}_p) - \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_l | e_\lambda \rangle$$
$$-(1-\delta_{kl}) J_{kl} \langle e_\lambda | (\boldsymbol{n}_l \cdot \boldsymbol{n}_k)(\boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_l) - (\boldsymbol{n}_l \cdot \boldsymbol{\sigma}_k)(\boldsymbol{n}_k \cdot \boldsymbol{\sigma}_l) | e_\lambda \rangle. \tag{15}$$

The diagonal part of (15) describes the energy change when the rotation is applied to single spins. The off-diagonal part corresponds to a simultaneous rotation of up to six spins.

It is convenient to express $|e_\lambda\rangle$ in terms of the basis states in the following way:

$$|e_\lambda\rangle = \sum_{\nu_1 \ldots \nu_6} \theta_{\nu_1 \ldots \nu_6} |\nu_1, \nu_2, \ldots, \nu_6\rangle \tag{16}$$

where $\nu_i = \pm 1$ (or $\uparrow, \downarrow$). We insert (16) into (15) and obtain

$$A_{kk}(\lambda) = \sum_{p \neq k} J_{kp} \sum_{\nu_1 \ldots \nu_6} \{[1-(n_k^z)^2] |\theta_{\nu_1 \ldots \nu_6}|^2 \nu_k \nu_p$$
$$+[1+(n_k^z)^2] \delta_{\nu_k - \nu_p} \theta^*_{\nu_1 \ldots -\nu_k, -\nu_p \ldots \nu_6} \theta_{\nu_1 \ldots \nu_6}\} \tag{17}$$

and for $k \neq l$

$$A_{kl}(\lambda) = -J_{lk} \sum_{\nu_1 \ldots \nu_6} \{(\boldsymbol{n}_k \cdot \boldsymbol{n}_l - n_k^z n_l^z) |\theta_{\nu_1 \ldots \nu_6}|^2 \nu_k \nu_l$$
$$+(\boldsymbol{n}_k \cdot \boldsymbol{n}_l + n_k^z n_l^z) \delta_{\nu_{k_1} - \nu_{k_l}} \theta^*_{\nu_1, \ldots -\nu_k - \nu_l \ldots \nu_6} \theta_{\nu_1 \ldots \nu_6}\}. \tag{18}$$

Notice that the sign of the diagonal part depends mainly on the exchange couplings and on the coefficients θ. The sign of the off-diagonal part, on the other hand, depends on the directions of rotations in a more complicated way. The stability against single-spin rotations is thus very easy to assess. Our results are as follows. In the ferromagnetic case the only local minimum is the ground state. However, in the spin-glass case, we have three local minima: the ground-state triplet $|E_0, 1\rangle$, the singlet $|E_1, 0\rangle$ and a quintuplet $|E_2, 2\rangle$ with $E_2 = -12.691$. (When $J_{ij} = -1$ the ground state is a singlet of energy -12 and three multiplets of energy -8 are excited local minima.)

It is interesting to ask whether the excited energy minima $|E_1, 0\rangle$ and $|E_2, 2\rangle$ are stable against multiple-spin rotations. As far as we know, a corresponding study for classical spin glasses has never been done. In order to answer the question, we have picked sets of random directions $\boldsymbol{n}_1, \ldots, \boldsymbol{n}_6$ and calculated the matrix elements of $A_{kl}(\lambda)$. The matrix was subsequently diagonalised in order to check for its positive definiteness. The quintuplet minimum $|E_2, 2\rangle$ was immediately found not to be stable with respect to this broader class of deviations. On the other hand, the singlet minimum $|E_1, 0\rangle$ is most likely to be a true local minimum even if six-spin deviations are taken into account. None of 100 000 sets of random directions considered gave rise to any instability. The system of figure 1 has thus two non-trivial local energy minima.

5. Other criteria

It is now interesting to ask if the local minima $|E_0, 1\rangle$ and $|E_1, 0\rangle$ are distinguished by anything else than a stability against spin rotations. We have thought of two such criteria.

The first one involves applying a random perturbation

$$\delta\mathcal{H} = \mu \sum_i \boldsymbol{B}_i \cdot \boldsymbol{\sigma}_i \tag{19}$$

to the Hamiltonian (1). In equation (19) μ denotes a magnetic moment and $\boldsymbol{B}_i$ are spatially random magnetic fields. We pick a state $|e_\lambda\rangle$ and calculate

$$T_\lambda = \sum_{\lambda' \neq \lambda} \langle e_\lambda | \delta\mathcal{H} | e_{\lambda'} \rangle^2. \tag{20}$$

T_λ is related to the probability of transitions out of $|e_\lambda\rangle$. It turns out that none of the states is distinguished by T_λ. If we restrict λ' in equation (20) to states of energies lower than e_λ then only the ground state is distinguished. With bigger systems, the densities of states come into play and it is possible to imagine that local energy minima could yield appreciably different transition rates than states away from a minimum.

Another idea involves studies of small oscillations, or spin waves, around the particular states $|e_\lambda\rangle$. The simplest approach is to take

$$\boldsymbol{\sigma}_i = \langle \boldsymbol{\sigma}_i \rangle + \delta\boldsymbol{\sigma}_i \tag{21}$$

eliminate the quadratic terms and derive an equation of motion in the form

$$\partial_t \delta\sigma_j^+ = -\mathrm{i} \sum_k B_{jk}^{(\lambda)} \delta\sigma_k^+ \tag{22}$$

where

$$B_{jk}^{(\lambda)} = -\langle \sigma_j^z \rangle J_{jk} + \delta_{jk} \sum_l \langle \sigma_l^z \rangle J_{jl}. \tag{23}$$

We have used the fact that $\langle \sigma_j^x \rangle = \langle \sigma_j^y \rangle = \partial_t \delta\sigma_j^z = 0$ and $\sigma_j^+ = \sigma_j^x + \mathrm{i}\sigma_j^y$. The average is in the eigenstate $|e_\lambda\rangle$.

The eigenvalues of $B_{jk}(\lambda)$ are the frequencies of spin waves excited around $|e_\lambda\rangle$. In the ferromagnetic ground-state multiplet the $S^z = 0$ state yielded all zero eigenvalues, whereas the $S^z = 1$ state gave rise to one zero and five negative eigenvalues. None of the eigenvalues was complex. The ground state, however, was in no way distinguished from most of the other states and the reality of the eigenvalues supplied no effective criterion to find a minimum.

For large systems, the classical ground state should work as a good 'medium' for exciting spin waves and then perhaps stable small oscillations would signify a true energy minimum. In the spin-glass case introducing local quantisation axes will be necessary to describe spin waves.

6. Conclusions

In this paper we have discussed various criteria for describing a local energy minimum in quantum spin systems. A proper criterion turned out to be investigating stability against small-spin deviations. This criterion coincides with the classical one even though states in the quantum and classical systems are defined differently. The system shown in figure 1 has three multiplets which are stable against single-spin rotations. The lower two of these persist to be local energy minima if multispin rotations are allowed. The role of such minima in the relaxation processes of quantum spin glasses remains to be elucidated.

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References

Banavar R J, Cieplak M and Muthukumar M 1985 *J. Phys. C: Solid State Phys.* **18** L157
Binder K and Young A P 1986 *Rev. Mod. Phys.* **58** 801
Bray A J and Moore M A 1980 *J. Phys. C: Solid State Phys.* **13** L655
Cieplak M and Łusakowski J 1986 *J. Phys. C: Solid State Phys.* **19** 5253
Edwards S F and Anderson P W 1975 *J. Phys. F: Metal Phys.* **5** 965
Fisher K H 1975 *Phys. Rev. Lett.* **34** 1438
Glauber R J 1963 *J. Math. Phys.* **4** 294
Orbach R 1959 *Phys. Rev.* **115** 1181
Theumann A 1986 *Phys. Rev.* B **33** 559
Walker L R and Walstedt R E 1980 *Phys. Rev.* B **22** 3816